

# Ising Simulations on the D-Wave QPU

Information Science & Technology Institute  
Rapid Response: D-Wave Effort Debrief

Mike Rogers XCP-8  
Robert Singleton XCP-8

6 October 2016

LA-UR-16-27649



# Outline

I. The Physics of the Ising Model

II. Embedding the Ising Model on a Chimera Graph

III. D-Wave Verification

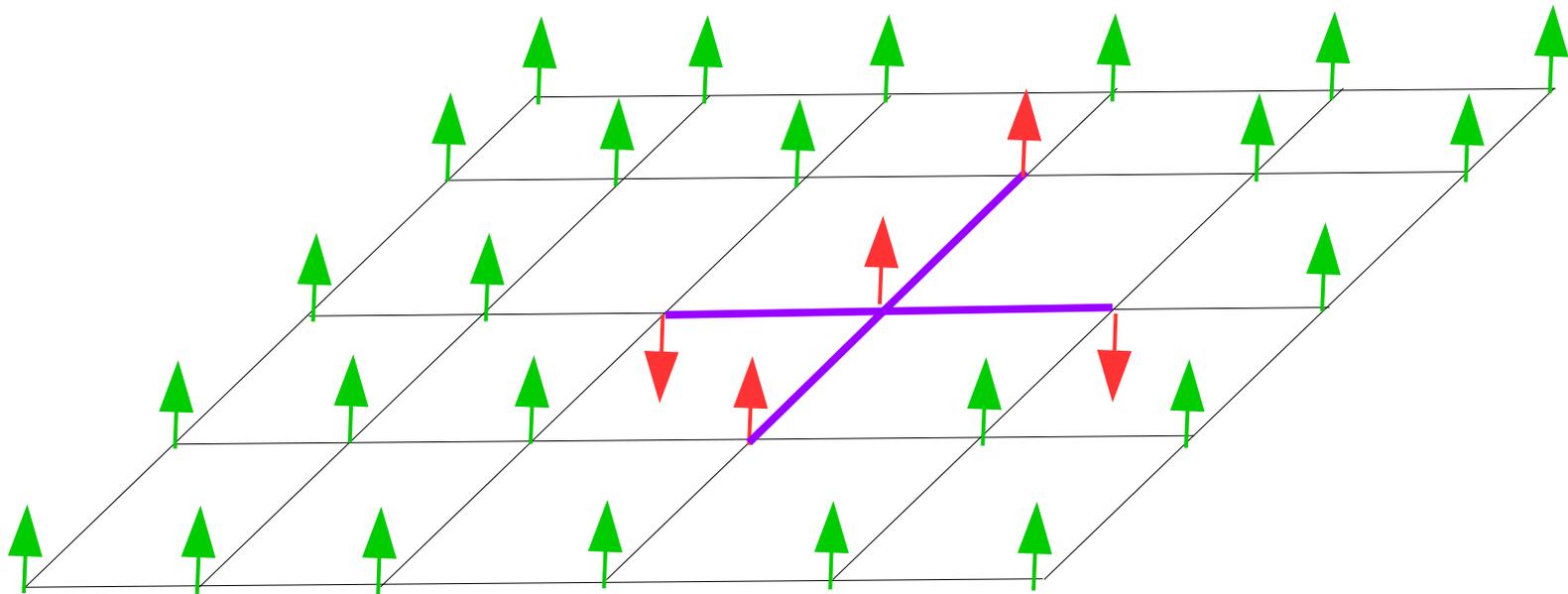
# I. The Physics of the Ising Model

$$H = \sum_i h_i q_i + \sum_{i < j} J_{ij} q_i q_j$$

$q_i = \pm 1$  spin at site  $i$

$J_{ij} =$  nearest neighbor spin-spin coupling

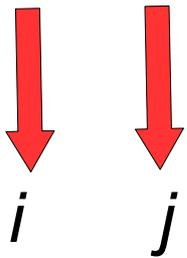
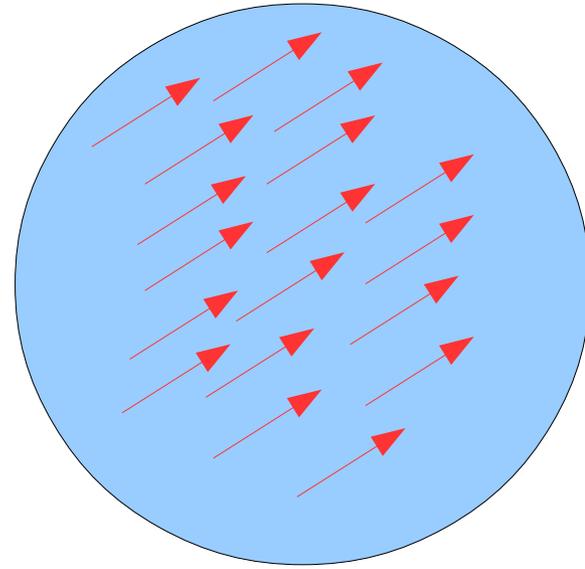
$h_i =$  coupling to external field



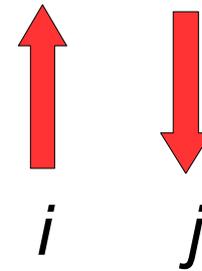
# I. The Physics of the Ising Model

Ferromagnet:  $J_{ij} < 0$

The spins want to align



have lower energy than

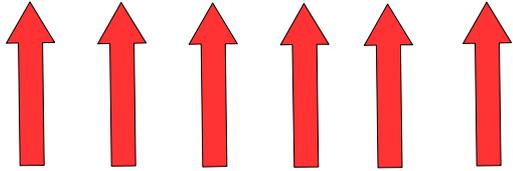


$$J_{ij}(-1)(-1) < 0$$

$$J_{ij}(+1)(-1) > 0$$

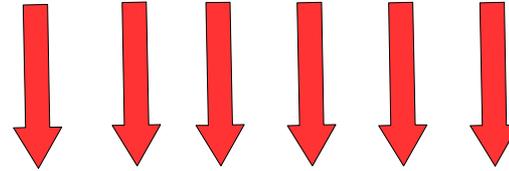
# I. The Physics of the Ising Model

At  $T=0$  there are two ground states: all sites are spin-up or all sites are spin-down



$$M = +1$$

positive magnetization per site



$$M = -1$$

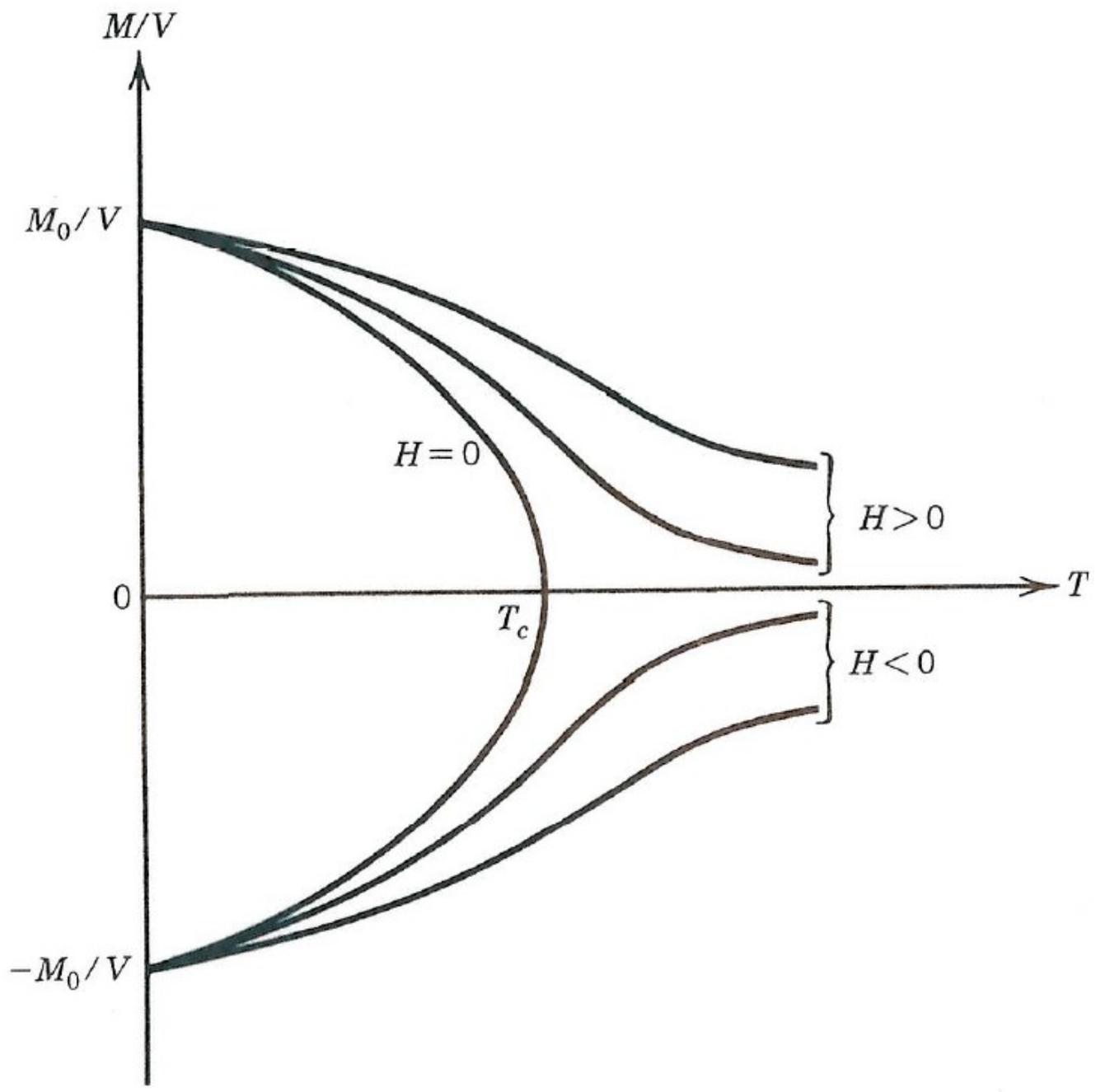
negative magnetization per site

In 2D  $T>0$  the spins fluctuate, and at high enough temperature the magnetization becomes zero. Spontaneous symmetry breaking! There is a phase transition at a critical temperature  $T_c$ .

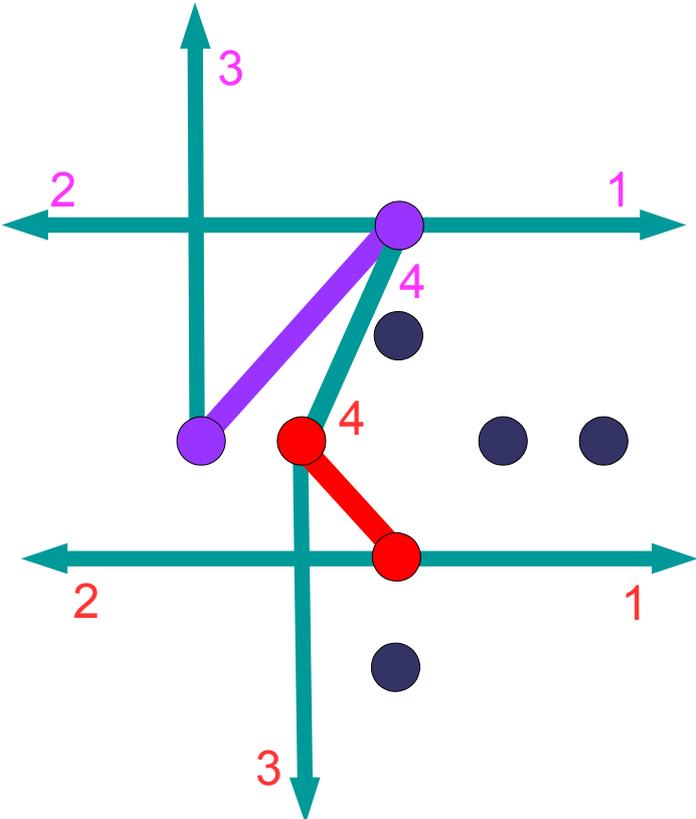
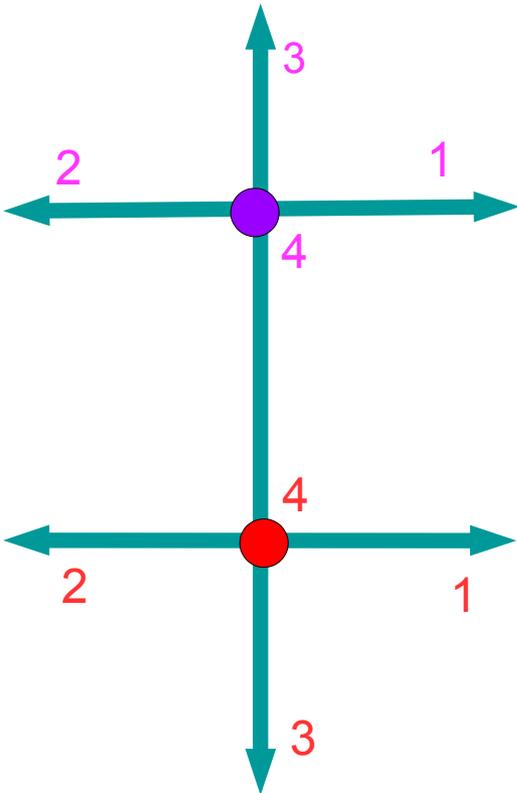
For the 2D square-lattice, the finite temperature partition function was calculated exactly [Onsager 1944].

The 1D partition function can also be calculated exactly, but there is no phase transition.

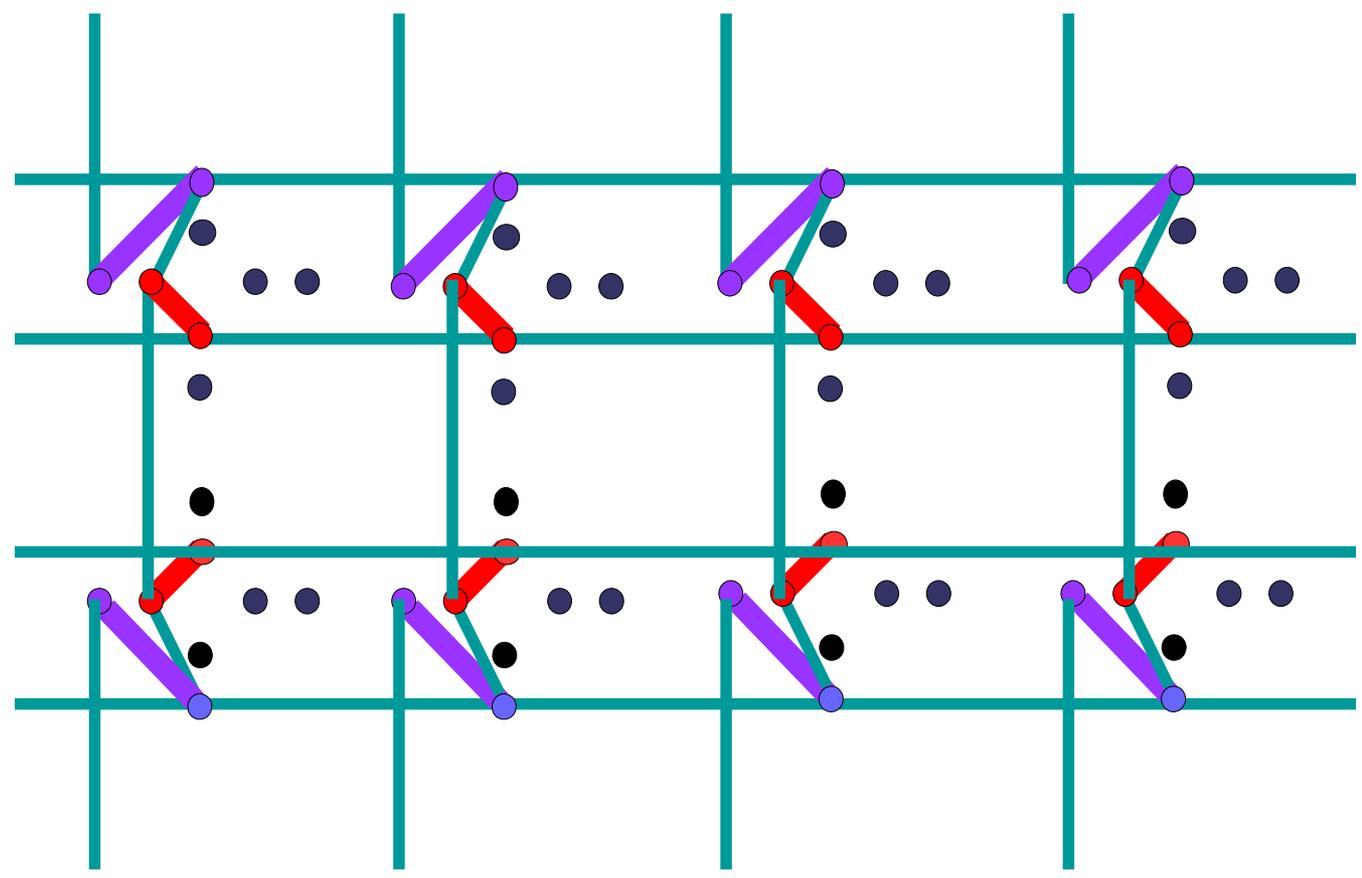
# I. The Physics of the Ising Model

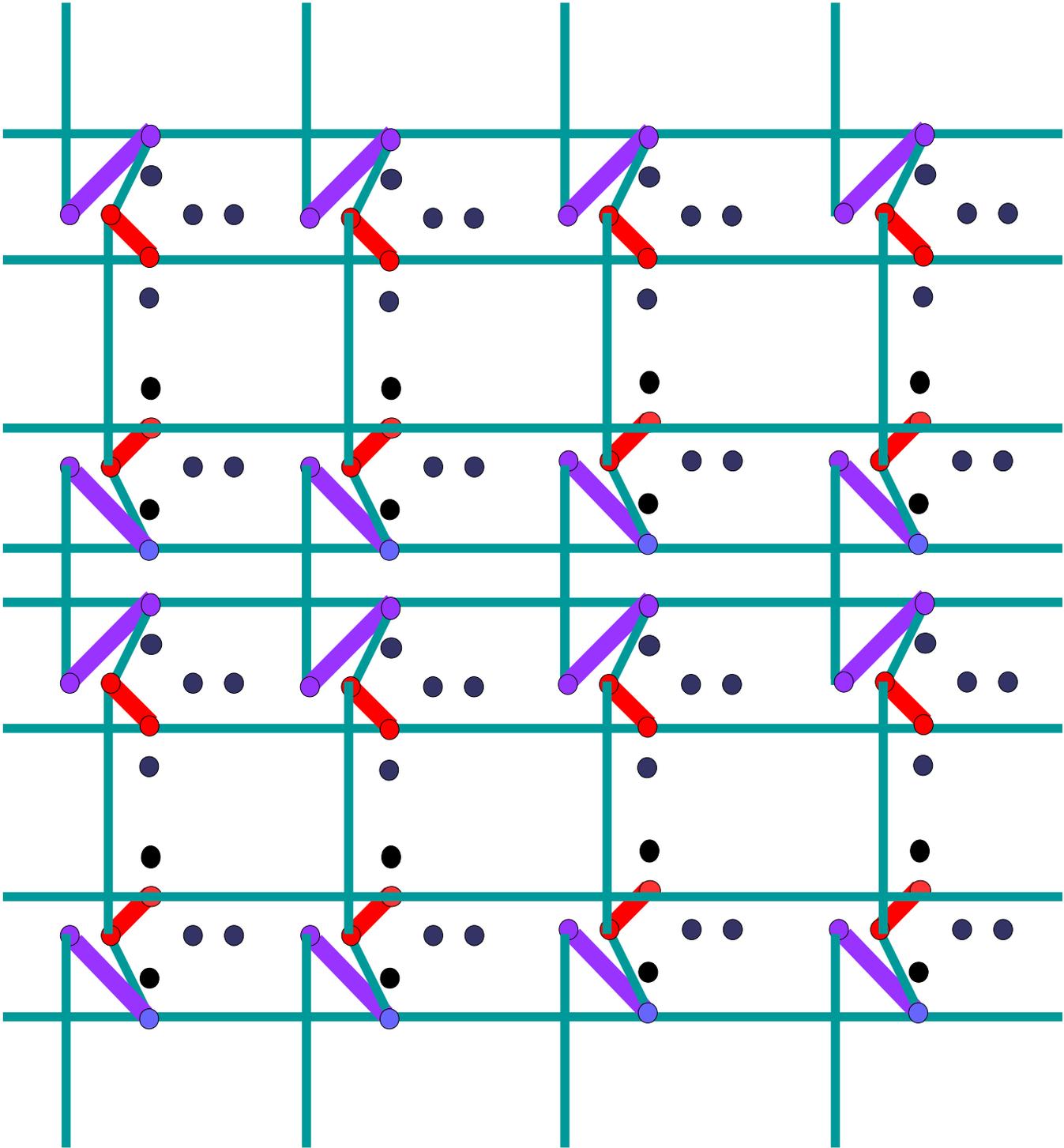


# II. Embedding 2D Ising model on the Chimera graph



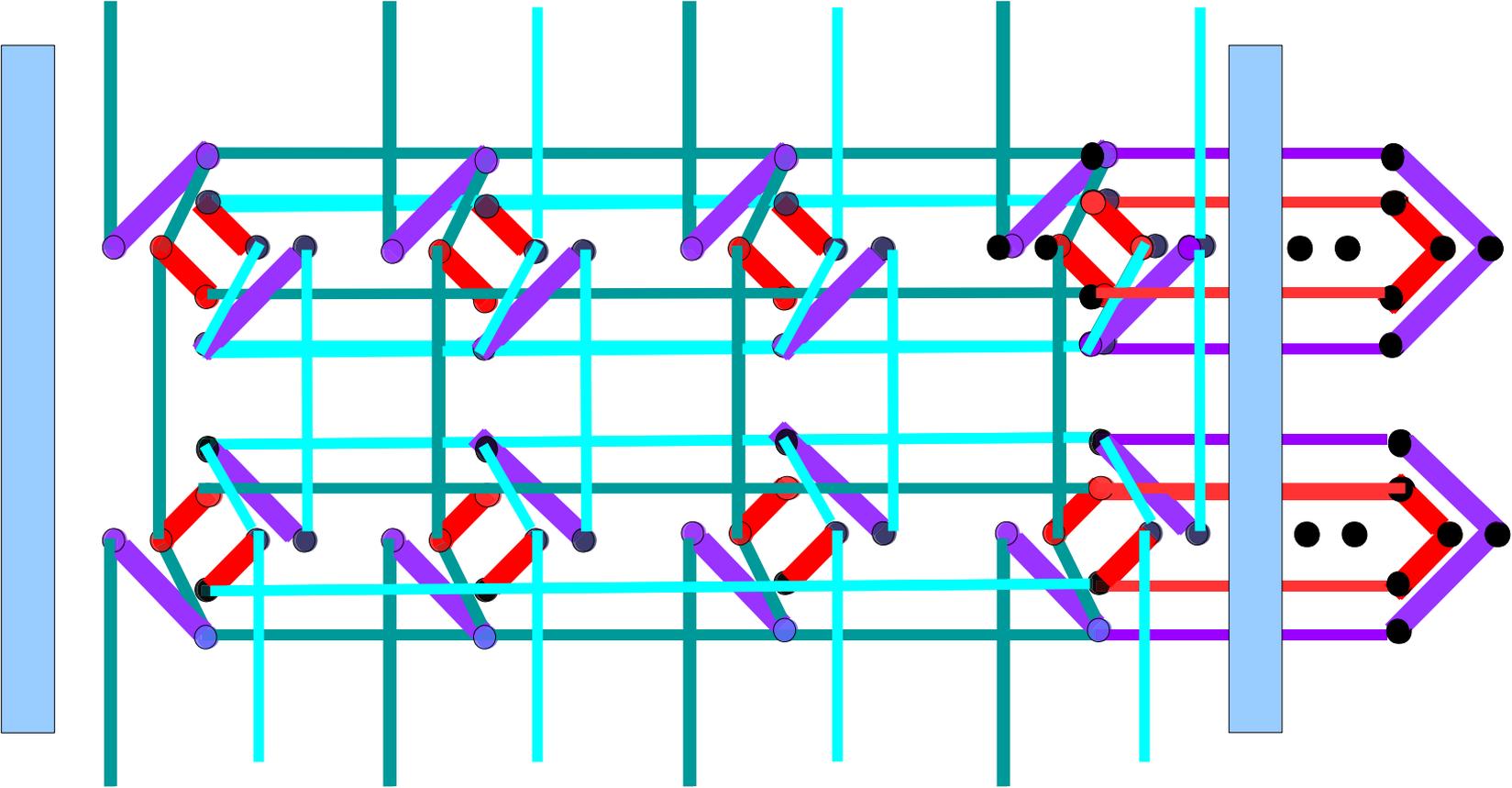
# II. Embedding 2D Ising model on the Chimera graph



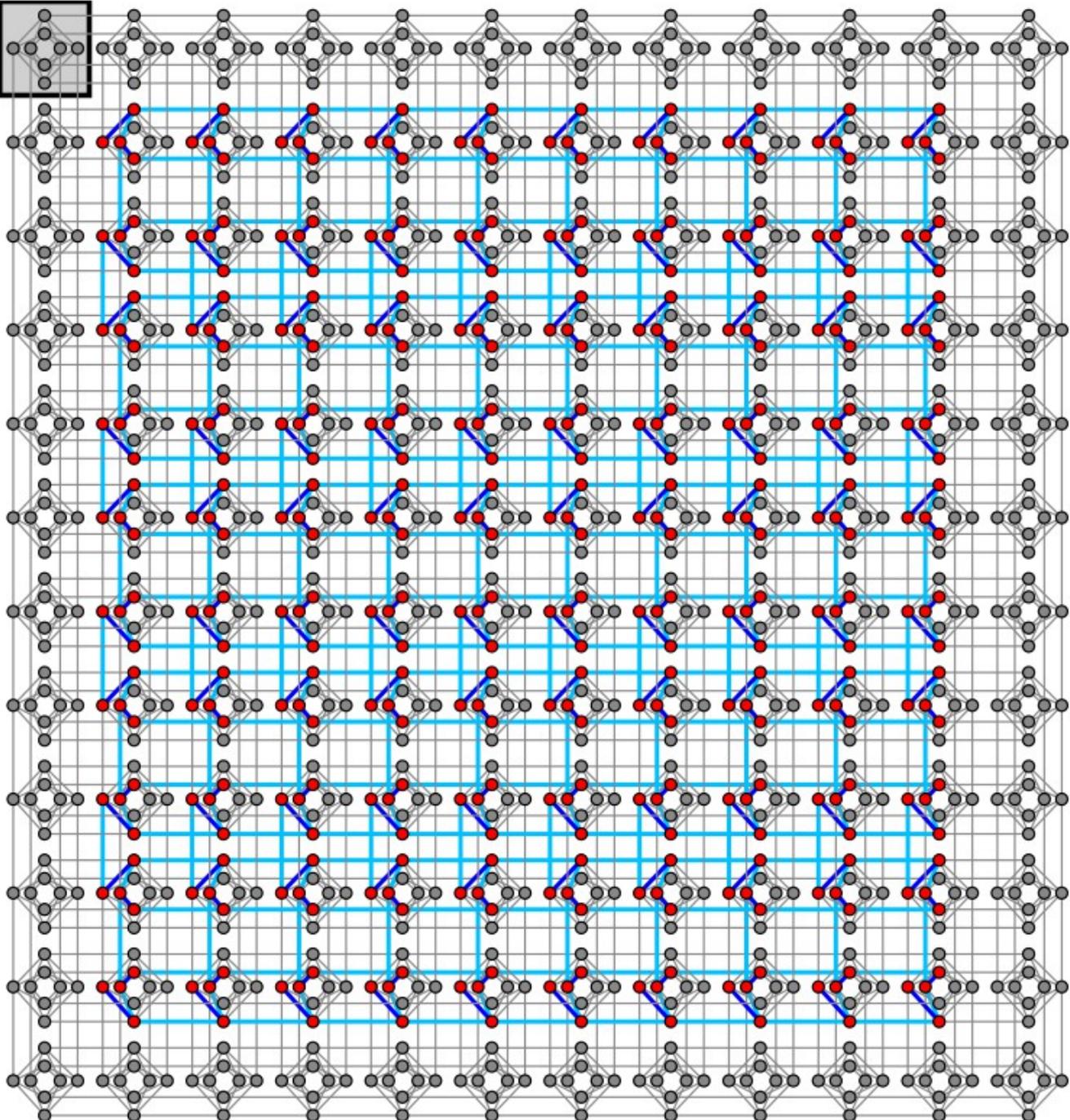


# II. Embedding 2D Ising model on the Chimera graph

Use both tilings for completely periodic BC's.



# II. Embedding 2D Ising model on the Chimera graph



## II. Embedding 2D Ising model on the Chimera graph

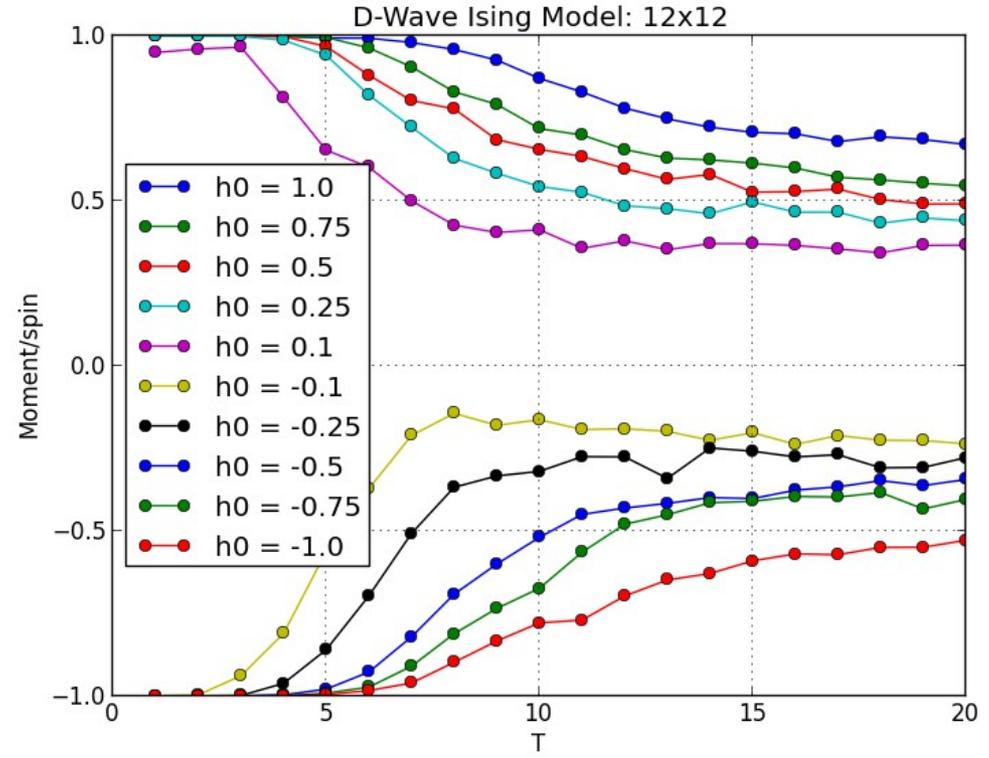
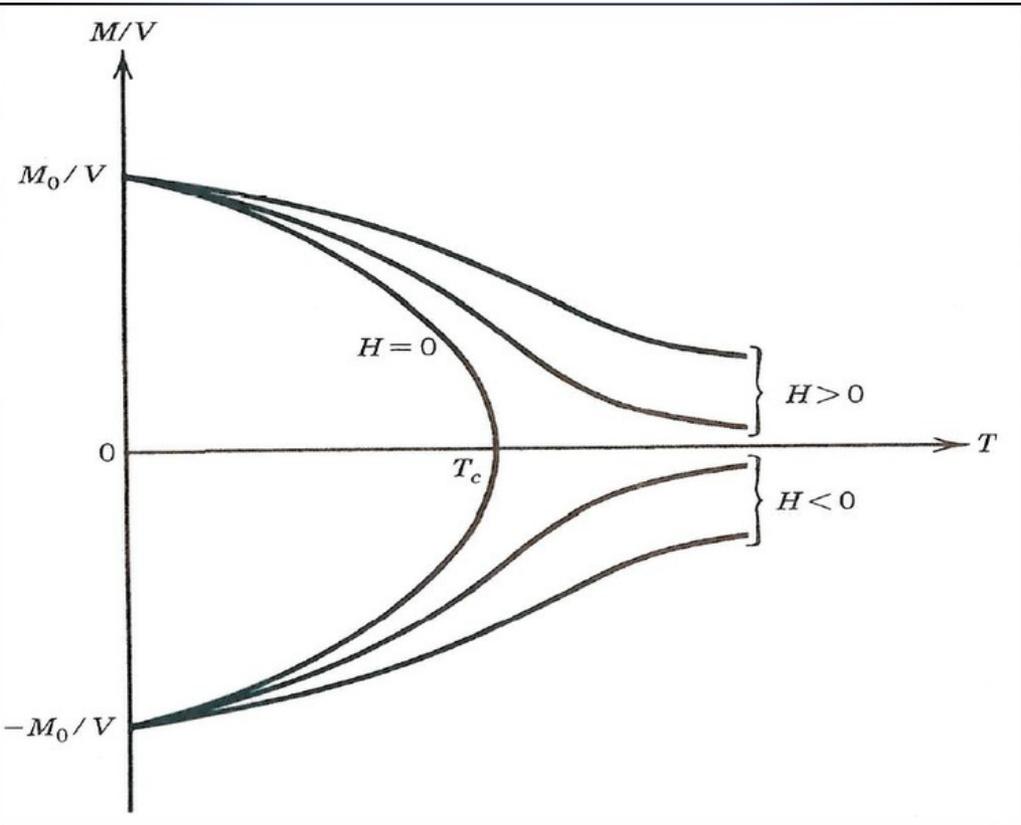
In practice, we used the SAPI embedding algorithm.

### III. D-Wave Verification

The 2D Ising model has an exact solution.

The D-Wave hardware **is** the algorithm.

We are comparing output to the exact solution, i.e. quantum verification.



### III. D-Wave Verification

The 1D Ising model also has an exact solution, but no phase transition